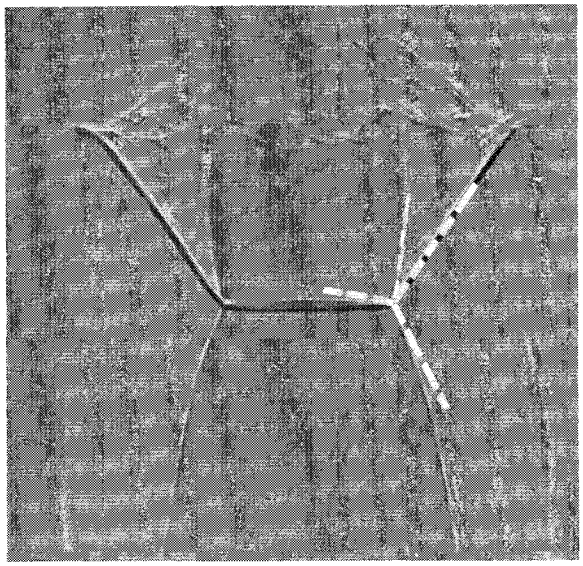

Fig. 1 Shadowgraph of an axially symmetric air jet flowing from a round orifice of 10 mm in diameter at a reservoir gage pressure of 70 psi (reproduced from Ref. 4). Superimposed are calculated slopes for the incident shock, reflected shock, Mach disk, and slip line locally at the triple point.

to the flow at the triple point. The photograph in Fig. 1 shows clearly the curvature of the Mach disk.

The triple-point results presented in Fig. 1 were based on interpolated estimates of the flow-field properties given by Ladenburg, Van Voorhis, and Winckler.[5] The Mach number used is 3.49 (which differs only slightly from the value of 3.51 used by Kawamura; we do not consider this an important difference as the triple-point solution, in general, is not as sensitive to the value of the incident Mach number as it is to the shock angles). The local flow angle used, assuming source flow for this particular example, is 8°. The calculated values for the shock angles appear to be in excellent agreement with the experimental values for the shock angles, in contradiction to the comments of Kawamura.[3]

It is believed that Kawamura failed to find valid triple-point conditions for this particular case because he assumed that the Mach disk was everywhere perpendicular to the axis of symmetry. At small pressure ratios, the Mach disk is small, the curvature is small, and the slope of the Mach disk at the triple point is much more nearly normal to the axis than it is at high-pressure ratios. As a result, Kawamura was able to obtain valid triple-point relations for these less underexpanded conditions by assuming that the shock was everywhere perpendicular to the axis.

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Liquid to Gas Heat Transfer in a Nuclear Reactor

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AMONG advanced high-temperature reactor concepts, the liquid core reactor has several advantages. However, heat-transfer rates obtainable by bubbling a propellant or working gas through a liquid are not particularly high. The purpose of this note is to point out another mechanism for transferring heat from a liquid to a gas which avoids the difficulties inherent in the bubbling process and might lead to a quite simple reactor design. Such a reactor might be ideal for use in magnetohydrodynamic (MHD) space power and propulsion cycles such as those described in Ref. 1.

Consider a gas flowing over the surface of a hot liquid with a velocity u. Assuming turbulent flow, the heat transfer by convection will be

$$q(\text{convection}) = N_{st}\rho_g u C_p[T(\text{wall}) - T(\text{gas})] \quad (1)$$

where N_{st} is the Stanton number, ρ_g is the gas density, C_p is the heat capacity, and T is the temperature. Now if the liquid is hot enough to have an appreciable vapor pressure, there will also be a transfer of mass from the liquid to the gas described by a formula similar to that just given for heat transfer. The vapor will carry with it its latent heat L_v, and if the vapor condenses into droplets as a consequence of being cooled by the gas, the net heat transfer from the liquid to the gas by this mechanism will be

$$q\ (\text{vaporization and condensation}) = N_{st}uL_v[\rho_v(\text{wall}) - \rho_v(\text{gas})] \quad (2)$$

where ρ_v (wall) is the vapor density in equilibrium with the liquid surface, and ρ_v (gas) is the vapor density in the gas. This mechanism of heat transfer to a gas by vaporization and subsequent condensation of a liquid is actually widespread in nature and in industry. The cooling tower is one common example, although the object in this case is to cool the liquid rather than to heat the gas.

In principle very large heat-transfer rates may be obtained in this manner. In particular, if the vapor pressure of the liquid exceeds the ambient pressure and boiling occurs, the rate will no longer be given by Eq. (2) but will certainly exceed it and increase rapidly. Violent boiling will probably lead to increased liquid entrainment in the reactor exhaust, and the limiting boiling rate will be at least partially determined by how much entrainment can be tolerated. In an MHD power or propulsion system, a relatively high rate might be tolerable, whereas in a direct cycle thermal rocket the rate must be very small.

If a very hot fissioning liquid is to be held in a container having a cooled liner there must be compatibility between the materials involved. Also, one must ask whether or not the heat loss involved in cooling the liner is reasonable. It is desirable that the liquid have a low heat conductivity and that the solid liner have a high one. One possible combination is liquid UO_2 in a tungsten liner.

Suppose as an example, that we wish to heat helium at 30 atm pressure to a temperature in the neighborhood of 4000° K. At this pressure it will take approximately 5000°K to make UO_2 boil. The resulting temperature profile through the liquid layer and container of the reactor is shown in Fig. 1. The profile is flat in the region where boiling occurs and falls off to the inner wall temperature, which in this case has been chosen to be 2500°K. The heat-flow equation in the region in which conduction to the wall of the container occurs is $d^2T/dx^2 = Q/\kappa$ where Q is the fission energy release

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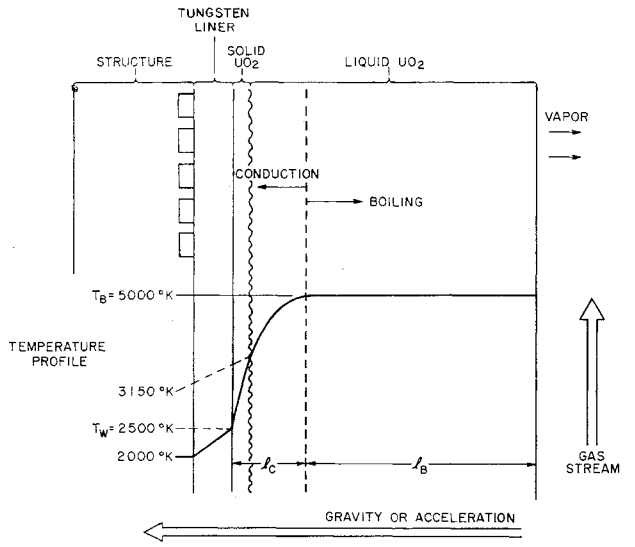

Fig. 1 Temperature profile through the liquid layer and container wall liner.

per second per unit volume, and κ is the thermal conductivity of the liquid. The thermal conductivity of UO_2 is 0.005 cal/sec/cm/°K at 2000°K. It will be assumed that it remains at this value up to 5000°K. The solution of this equation with appropriate boundary conditions yields

$$q(\text{wall}) = [2\kappa Q(T_B - T_W)]^{1/2} \quad (3)$$

and the thickness of the fuel region in which conduction to the wall occurs is

$$l_c = \{[2\kappa(T_B - T_W)]/Q\}^{1/2} \quad (4)$$

For the temperatures shown in Fig. 1 and for $Q = 1$ kw/cm³,

$$q(\text{wall}) = 320 \text{ w/cm}^2 \quad (5)$$

$$l_c = 0.32 \text{ cm} \quad (6)$$

and if we choose 2000°K as the temperature of the outer surface of the liner, the liner thickness can be

$$l_w = 1.83 \text{ cm} \quad (7)$$

assuming it is tungsten with a thermal conductivity of 0.28 cal/cm °K sec. If the total thickness of liquid is 10 cm, the heat removed by boiling must be

$$q(\text{boiling}) = l_B Q \approx 10 \text{ kw/cm}^2 \quad (8)$$

On comparing q(wall) with q(boiling), it can be seen that the heat lost to the container is only a small fraction of the heat delivered by boiling, and its magnitude is such that it can easily be handled by conventional cooling techniques. This result assumes that within the liquid layer there is no convection, natural or otherwise. However, since the liquid is being cooled from below, natural convection should not occur.

For space applications, the reactor configuration might take the form of an externally moderated cavity with the liquid fuel held in a rotating drum similar to the configurations that have been proposed for dust and (bubbling) liquid core reactors (i.e., Refs. 2 and 3).

As an example, for the conditions previously cited, the delivery of 100 Mw of heat would require one square meter of liquid surface. This could be provided in a drum approximately one-half meter in diameter and one-half meter long. Since the heat load on the liner goes only as the square root of Q, it should be possible to use much higher rates of heat release if so desired.

In the preceding example, if we assume that all condensed uranium dioxide droplets are spun out of the gas before it leaves the reactor at 4000°K, then the UO_2 remaining in vapor form will constitute about 3 mole% of the exhaust gas stream. As a result of the difference in molecular weight, the weight flow rate of UO_2 will be approximately equal to the weight flow rate of helium. This is clearly unsuitable for a thermal rocket but could be tolerated in an MHD power cycle.

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Constraint Surface Normals for Structural Synthesis Techniques

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MANY modern techniques for the optimum synthesis of structures are based upon the concept of a design space. In an S-dimensional design space, each Cartesian coordinate axis represents a design variable, and thus a point in this space $\mathbf{D} = (d_1, d_2, \ldots, d_S)$ represents a design. Furthermore, each design has associated with it a value of the objective function (weight, cost, etc.) and has a behavior as determined by the application of an analysis method. In the typical synthesis problem the design corresponding to the minimum of the objective function is sought subject to inequality constraints on the behavior and perhaps on the design variables themselves.

In most realistic structural problems the functions defining the constraint surfaces cannot be expressed explicitly in terms of the design variables because the performance or behavior quantities (stress, displacement, etc.) themselves cannot be expressed explicitly in terms of the design variables. This fact has made the application of some of the more advanced nonlinear mathematical programing techniques[1–3] difficult if not impossible because a common feature of these highly directed methods is a need for the normal vectors to the locally active constraint surfaces. In some cases[4] these vectors have been obtained by the application of a finite difference scheme in which a "star" of nearby designs is analyzed. The setting up and solution of these "perturbed" designs is a time consuming and costly operation, and much structural synthesis work (e.g., Refs. 5 and 6) has sought to develop efficient methods of moving about in the design space without the benefit of this knowledge of the constraint surface orientation.

This note proposes a simple method for obtaining the normals to the constraint surfaces in structural (or other) synthesis problems where a linear analysis method is appropriate for predicting the behavior of the structure. Under certain common conditions the components of the normals so calculated will be exact; otherwise they will be approximate but are obtained with considerably less effort than by complete finite difference and are likely to be more accurate.

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